

# Prognostics of polygonalization of high-speed railway train wheels using a generalized additive model smoothed by spline-backfitted kernel

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**Abstract**— A method for the prognosis of polygonalization in high-speed railway train wheels is developed based on a generalized additive model. Unlike most previous studies, this study uses field data, so findings can help improve practical maintenance efficiency. A spline-backfitted kernel is used to improve computation efficiency when figuring out model parameters. This prognostics method can be applied to practical railway management decisions.

**Keywords**—*prognostics, preventive maintenance, generalized additive model, spline-backfitted kernel, polygonalization, high-speed railway, train wheels*

## I. INTRODUCTION

Polygonalization is a common periodic wheel defect [1] found on the wheels of locomotives, metro trains, ordinary trains and high-speed trains [2][3]. Polygonalization significantly decreases the safety and availability of high-speed railway (HSR) trains. In 1998, in Germany, an inter-city express (ICE), a kind of HSR train, suffered a catastrophic accident because of polygonalization [1]. However, polygonalization can only be detected by out-of-roundness detection machines; this type of wheel deformation cannot be seen by humans. Therefore, in practical operation, to detect polygonalization, HSR trains are raised, as shown in Fig.1, and each wheel is diagnosed in turn, significantly decreasing the availability of the trains. Until now, maintenance plans to decrease the occurrence rate of polygonalization has focused on scheduled maintenance based on operating experience. However, scheduled maintenance wastes large amounts of resources in most conditions and fails to restore polygonalization in time in some circumstances. A prognostics method is necessary to ensure effective preventive maintenance and improve maintenance efficiency.



Fig. 1. HSR train maintenance workshop

Most studies on the polygonalization of train wheels are based on physical or mathematic models [4][5][6][7][8][9]. But the mechanism of the formation of polygonalization is too complex to be described by a model. The impact of the operating environment makes this especially difficult. Some studies have discussed mechanisms and correlated phenomena using physics models. Data-driven approaches have been used to study the reliability of train vehicle wheels [10]. Field data have been used to analyze the reliability of locomotive wheels [11][12], and the location of wheels has been found to be connected with hazard rate [13][14]. In our study, field data on polygonalization are used to improve maintenance efficiency.

A generalized additive model (GAM) has the interpretability advantages of a generalized linear model (GLM), in that the contribution of each independent variable to the prediction is clearly encoded. However, it has substantially more flexibility because the relationships between independent and dependent variables are not assumed to be linear. When a regression model is additive, the interpretation of the marginal impact of a single variable does not depend on the values of the other variables in the model. Hence, by simply looking at the output of the model, we can make simple statements about the effects of the predictive variables.

In our study, a generalized additive model is adapted to prognosticate polygonalization. For the last two decades, research has focused on the additive model popularized by Hastie and Tibshirani[15]. In later work, polynomial spline estimators were introduced into the model [16], and the estimators were further extended to weakly dependent data [17]. Hastie and Tibshirani [15] proposed backfitting estimators but did not provide theoretical justifications. Nielsen implemented a modified backfitting algorithm and called it the smooth backfitting estimator [18]. A two-step procedure was suggested by Fan for local quasi-likelihood estimation [21]. Because of its computational intensity, kernel estimation is rarely used for high dimensions. However, with the use of spline-backfitted kernel (SBK) smoothing, univariate nonparametric regression can be extended to a generalized additive model (GAM) [19][20]. SBK smoothing has also been developed for a much simpler additive model [21][22][23]. The spline method is fast, but there is no pointwise confidence interval or consistency in GAM.

Therefore, the spline-backfitted kernel is a good choice to balance estimation accuracy and computational efficiency.

This paper formulates an efficient method to predict the polygonalization of HSR train wheels using a generalized additive model while considering the wheels' profile, location, operating environment, maintenance records and total distance traveled. The article is organized as follows. Section 2 introduces the formulation of the generalized additive model. Section 3 uses spline-backfitted kernel estimation to solve the prediction model. Section 4 conducts a case study of the wheels on China's HSR trains. Section 5 draws conclusions and suggests future work.

## II. MODEL FORMULATION

Based on the Cox proportional hazard model, a flexible prognostic model is formulated by fusing GAM as follows:

$$y = \sum_{i=1}^I s_i(x_i) \quad (1)$$

where  $S_i(\cdot)$  is the spline function for covariate  $x_i$ , and  $y$  is a dependent variable. The covariates are wheel profile indexes and operating distance in our model. The wheel profile indexes include diameter, height of flange, thickness of flange and diameter difference of wheels located on the same axle.

This study proposes a partial logistic model for ungrouped data with distance-dependent covariates, written as

$$\ln y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n \quad (2)$$

The linear predictor can be replaced by a sum of smooth functions  $S_i(x_i)$ , such that

$$\ln y = \beta_0 + s_0(x_0) + s_1(x_1) + s_2(x_2) + \dots + s_n(x_n) \quad (3)$$

And the smooth functions can be expressed as

$$\begin{aligned} s_0(t) &= \sum_{j=1}^{q_1} \beta_0 b_{0j}(t) \\ s_1(x) &= \sum_{j=1}^{q_1} \beta_{q_0+j} b_{1j}(x) \\ &\dots \\ s_I(x) &= \sum_{j=1}^{q_I} \beta_{q_0+q_1+\dots+q_{I-1}+j} b_{Ij}(t) \end{aligned} \quad \square(4)$$

where  $q_1, q_2, \dots, q_I$  are the numbers of knots, and  $b_{ij}(x)$  is the B-spline basis function. Then, the parameters can be estimated by maximizing the partial log-likelihood.

## III. PARAMETER ESTIMATION

### A. Spline-Backfitted Kernel (SBK)

The 'backfitting' technique was first proposed by Hastie and Tibshirani (1986,1990) to estimate parameters in GAM. The advantage of 'backfitting' is its ability to use almost any smoothers or regression modeling techniques to represent the component function in GAM; its disadvantage is the difficulty figuring out the degree of smoothness. In general, it is a perfect and concise method to fit GAM with smoothers, some of which are not really smooth, representing component functions. The principle of backfitting is to estimate all the smooth components in an additive model by iteratively decreasing partial residuals from the model. The partial residuals of the  $j$ th smooth term result from the residuals of all the smooth terms except for the  $j$ th term estimated from response variables, which can be represented as

$$y_j = \alpha + \sum_{j=1}^m f_j(x_{ji}) + \epsilon_i \quad (5)$$

where  $f_j$  is the  $j$ th smooth component function,  $x_{ji}$  represents the  $i$ th covariate of the  $j$ th term, and  $\hat{f}_j$  is the estimation of  $f_j(x_{ji})$ . The pseudo algorithm of the backfitted-spline kernel can be noted as the following:

1. Set  $\hat{f}_j = 0$ , for  $j=1, \dots, m$
2. Repeat step 3 to 5 until  $\hat{f}_j$  stops changing
3. For  $j=1, \dots, m$ , repeat steps 4 and 5
4. Figure out partial residuals  $e_p^j = y - \hat{\alpha} - \sum_{k \neq j} \hat{f}_k$

- Set  $\hat{f}_j$  equal to the result of smoothing  $e_p^j$  with respect to  $x_j$

The advantage of backfitting is obvious: it gives a wide variety of possible smoothing methods to estimate component function. However, the calculation efficiency of GCV or CV is not satisfactory,  $O(mn^2)$ , where  $n$  is the number of data.

### B. Spline-Backfitted Kernel Estimation

The unknown functions  $\{m_\alpha(x_\alpha)\}_{\alpha=2}^d$  and constants  $c$  can be pre-estimated by linear splines, and these estimates can be used to construct the SBK estimator. Let  $0 = \xi_0 < \xi_1 < \dots < \xi_N < \xi_{N+1} = 1$  denote a sequence of equally spaced points, as well as interior knots, on  $[0, 1]$ . The subinterval  $[\xi_J, \xi_{J+1}]$  is  $H = (N+1)^{-1}$ , where  $0 \leq J \leq N$ . Denote the degenerate knots by  $\xi_{-1} = 0$ ,  $\xi_{N+2} = 1$ .

Define the linear B-spline basis as

$$b_j(x) = (1 - |x - \xi_j| / H)_+ = \begin{cases} (N+1)x - J + 1, & \xi_{j-1} \leq x \leq \xi_j \\ J + 1 - (N+1)x, & \xi_j \leq x \leq \xi_{j+1} \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

□ Denote  $\alpha$  as the space of empirically centered linear spline functions on  $[0, 1]$ , written as

$$G_{n,\alpha}^0 = \left\{ g_\alpha : g_\alpha(x_\alpha) = \sum_{j=0}^{N+1} \alpha_j b_j(x_\alpha), E_n \{g_\alpha(x_\alpha)\} = 0 \right\} \quad (7)$$

Define the log-likelihood function as

$$\hat{L}(g) = n^{-1} \sum_{i=1}^n [Y_i g(X_i) - b\{g(X_i)\}], \quad g \in G_n^0 \quad (8)$$

which has a unique maximizer. The multivariate function  $m(x)$  is now estimated using the additive spline function  $\hat{m}(x) = \arg \max_{g \in G_n^0} \hat{L}(g)$

## IV. CASE STUDY

In this section, we study field data of Chinese HSR train wheels, including over 3000 wheels and 25000 records. We randomly choose 95 percent as training data and five percent as validation data. We use the following figures to explain the correlation between variates and degree of polygonalization.

The blue lines in the following figures show the fitting of the generalized additive model; the red line shows a 95 percent

confidence interval. Fig.2 shows a sharp decline in the degree of polygonalization when the diameter of the wheel reaches about 840 mm; this means the polygonal wear rate sharply increases for those wheels with a diameter less than 840 mm.

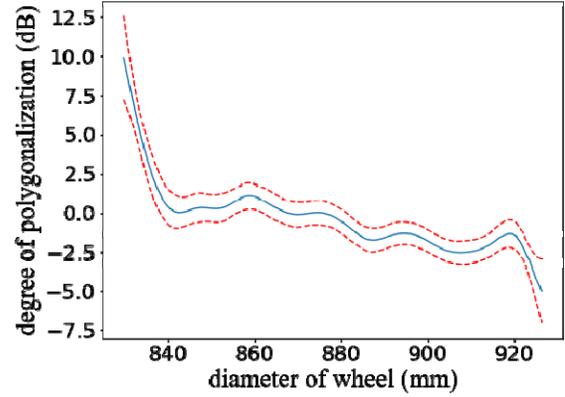


Fig. 2. Correlation between diameter and polygonalization

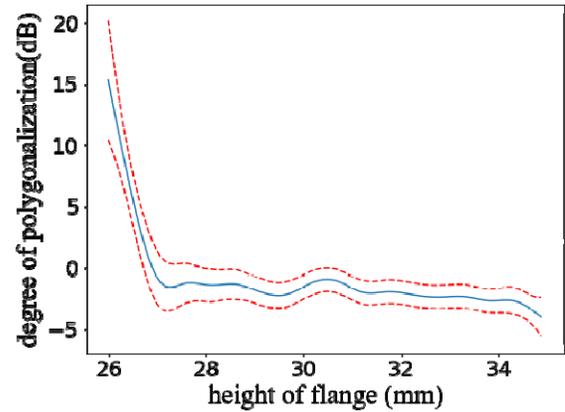


Fig. 3. Correlation between height of flange and polygonalization

Fig.3 shows a sharp decline in the degree of polygonalization when the height of the flange of the wheel reaches 27 mm; this means the polygonal wear rate sharply increases for wheels with a diameter less than 27 mm.

For other wheel profile indexes, the model shows little significant correlation, as indicated in the following figures.

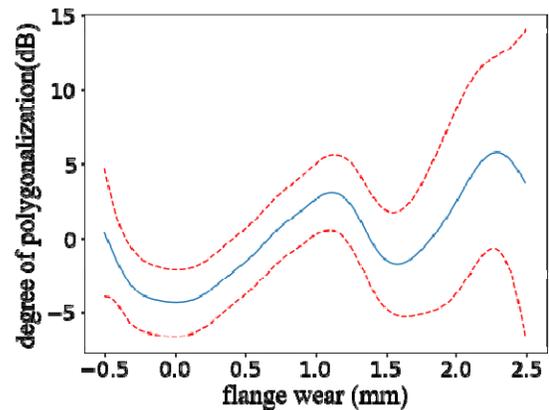


Fig. 4. Correlation between flange wear and polygonalization

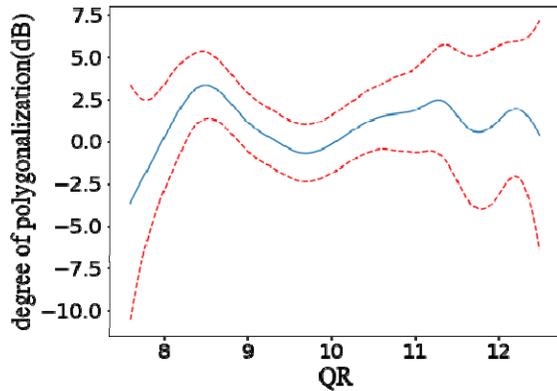


Fig. 5. Correlation between QR and polygonalization

In addition to wheel profile characteristics, we also consider operating distance of the wheels. As Fig.6 shows, the polygonal wear rate gradually increases up to about 190,000 km. After that point, there is a sharp increment in polygonalization.

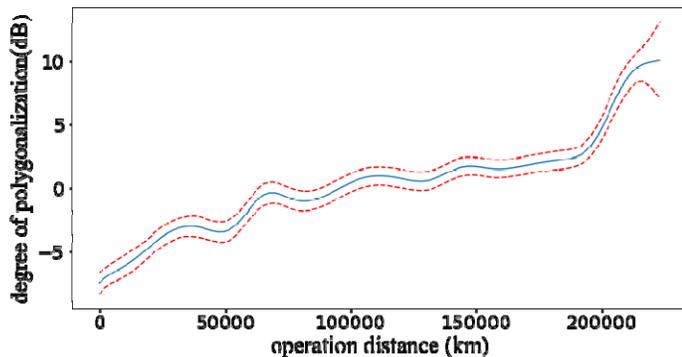


Fig. 6. Correlation between operation distance and polygonalization

The R-squared value of the model is 0.43 compared with 0.32 of the generalized linear model when fitting the same data set. Three groups of validation data randomly chosen from the data set are used to testify prognostics performance of the model. The negative positive rate (NPR) and the positive negative rate (PNR) of polygonalization prediction given by GLM and GAM have been listed in the following table. From the result, we can find that both NPR and PNR given by GAM are lower than those given by GLM. Therefore, GAM performs better in prognostics in this case study. The prognostics performance of GAM is good enough to be considered as a decision reference for polygonalization maintenance.

TABLE I. PROGNOSTICS PERFORMANCE

	GLM		GAM	
	NPR	PNR	NPR	PNR
Set 1	0.23	0.16	0.12	0.08
Set 2	0.24	0.18	0.11	0.11
Set 3	0.27	0.16	0.15	0.09

The case study indicates that the generalized additive model is good at describing changes in trends by using splines, thus helping researchers analyze the data more efficiently. We also find the addition of the spline-backfitted kernel makes computation much faster than previous generalized additive methods.

## V. CONCLUSION

A method for the prognosis of polygonalization of HSR wheels is proposed based on the generalized additive model. The spline-backfitted kernel makes data fitting smoother and faster. In a study using field data, the model shows reliable performance. The proposed method can be used to simultaneously improve maintenance efficiency and operation safety. In the future, a combination of splines and tensors applied in GAM will be considered to improve fitting performance. More analysis of field data will be conducted using the model.

## REFERENCES

- [1] D. W. Barke and W. K. Chiu, "A review of the effects of out-of-round wheels on track and vehicle components," *Proceedings of the Institution of Mechanical Engineers Part F: Journal of Rail & Rapid Transit*, 219(3):151-175, 2005.
- [2] G. Tao, L. Wang, Z. Wen, et al, "Measurement and assessment of out-of-round electric locomotive wheels," *Proceedings of the Institution of Mechanical Engineers Part F Journal of Rail & Rapid Transit*, 232(1): 275-287, 2018.
- [3] S. Iwnicki, *Handbook Of Railway Vehicle Dynamics*. Boca Raton, FL: CRC Press, 2006, 23-45.
- [4] X. Y. Liu and W.M. Zhai, "Analysis of vertical dynamic wheel/rail interaction caused by polygonal wheels on high-speed trains," *Wear*, 314: 282-290, 2014.
- [5] X. W. Wu, S. Rakheja, A. K.W. Ahmed, et al, "Influence of a flexible wheelset on the dynamic responses of a highspeed railway car due to a wheel flat," *Proceedings of the Institution of Mechanical Engineers Part F: Journal of Rail & Rapid Transit*, 232(4): 1033-1048, 2018.
- [6] R. Lewis and U. Olofsson, *Wheel-Rail Interface Handbook*. Elsevier, 2009, 32-65.
- [7] E. Brommundt, "A simple mechanism for the polygonalization of railway wheels by wear," *Mechanics Research Communications*, 24(4), 435-442, 1997.
- [8] W. Chen, "Research on wheel polygonalization of High speed train." not published.
- [9] G. Tao, L. Wang, Z. Wen, Q. Guan, X. Jin, "Experimental investigation into the mechanism of the polygonal wear of electric locomotive wheels," *Vehicle System Dynamics*, 56(1), 1-17, 2018.
- [10] J. Lin and M. Asplund, "A comparison study for locomotive wheels' reliability assessment using the weibull frailty model," *Journal of Eksploatacja i Niezawodnos - Maintenance and Reliability*, 16(2): 276-287, 2014.
- [11] J. Lin and M. Asplund, "Bayesian semi-parametric analysis for locomotive wheel degradation using gamma frailties," *Institution of Mechanical Engineers Proceedings Part F: Journal of Rail and Rapid Transit*, 229(3): 237-247, 2015.
- [12] J. Lin, J. Pulido, M. Asplund, "Reliability analysis for preventive maintenance based on classical and bayesian semi-parametric degradation approaches using locomotive wheel-sets as a case study," *Reliability Engineering & System Safety*, 134:143-156, 2015.
- [13] J. Lin, M. Asplund, A. Parida, "Reliability analysis for degradation of locomotive wheels using parametric bayesian approach," *Journal of Quality and Reliability Engineering International*, 30 (5): 657-667, 2014.

- [14] B. Liu, J. Lin, L. Zhang, M. Xie, "A dynamic maintenance strategy for prognostics and health management of degrading systems: application in locomotive wheel-sets," 2018 IEEE International Conference on Prognostics and Health Management.
- [15] T. J. Hastie and R. J. Tibshirani, *Generalized Additive Models*. London: Chapman and Hall, 1986, 132-145.
- [16] L. Xue and L. Yang, "Additive coefficient modeling via polynomial spline," *Statistica Sinica*, 16:1423–1446, 2006.
- [17] J. P. Nielsen and S. Sperlich, "Smooth backfitting in practice," *Journal of the Royal Statistical Society: Series B*, 67:43–61, 2005.
- [18] O. B. Linton, "Efficient estimation of additive nonparametric regression models," *Biometrika*, 84:469–473, 1997.
- [19] J. Wang and L. Yang, "Efficient and fast spline-backfitted kernel smoothing of additive models," *Annals of the Institute of Statistical Mathematics*, 61(3):663-690, 2009.
- [20] R. Liu, L. Yang, W.K. Hurdle, "Oracally efficient two-step estimation of generalized additive model," *Journal of the American Statistical Association*, 108(502):619-631, 2013.
- [21] Q. Song and L. Yang, "Oracally efficient spline smoothing of nonlinear additive autoregression models with simultaneous confidence band," *Journal of Multivariate Analysis*, 101(9):2008-2025, 2010.
- [22] L. Yang and R. Liu, "Spline-backfitted kernel smoothing of additive coefficient model," *Econometric Theory*, 26(1):29-59, 2010.
- [23] L. Wang and L. Yang, "Spline estimation of single-index models," *Statistica Sinica*, 19(2):765-783, 2009.